

Yukawa Interactions and Supersymmetric Electroweak Baryogenesis

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We analyze the quantum transport equations for supersymmetric electroweak baryogenesis including previously neglected bottom and tau Yukawa interactions and show that they imply the presence of a previously unrecognized dependence of the cosmic baryon asymmetry on the spectrum of third generation quark and lepton superpartners. For fixed values of the CP-violating phases in the supersymmetric theory, the baryon asymmetry can vary in both magnitude and sign as a result of the squark and slepton mass dependence. For light, right-handed top and bottom quark superpartners, the baryon number creation can be driven primarily by interactions involving third generation leptons and their superpartners.

If the universe was matter-antimatter symmetric at the end of the inflationary epoch then the microphysics of the subsequently evolving cosmos must have dynamically generated the cosmologically observed baryon asymmetry (the baryon to entropy density ratio n_B/s),

$$8.36 \times 10^{-11} < n_B/s < 9.32 \times 10^{-11} (95\% \text{ C.L.}) \quad (1)$$

[2]. The Standard Model (SM) of particle physics and standard big bang cosmological model contain all the necessary ingredients (Sakharov criteria [1]) for successful baryogenesis: baryon number violation, charge conjugation (C) and charge conjugation-parity (CP) violation, and departures from thermal equilibrium. However, fundamental symmetry tests and collider constraints indicate that generating the observed n_B/s requires physics beyond the SM (see *e.g.* [3]).

Among most widely studied viable possibilities is electroweak baryogenesis (EWB) which is testable with low-energy searches for permanent electric dipole moments (EDMs) and high-energy studies at the Large Hadron Collider (LHC) [4]. In this scenario, electroweak symmetry-breaking (EWSB) – a cosmological transition in which $SU(2)_L$ is broken at temperature $T \sim 100$ GeV – proceeds via a strong, first order phase transition during which bubbles of broken electroweak symmetry nucleate and expand in a background of unbroken symmetry. Particle-antiparticle asymmetries generated by CP-violating interactions at the bubble wall induce a non-zero density of left-handed fermions, n_{left} , that diffuses into the unbroken background where baryon number violating $SU(2)_L$ sphaleron (electroweak sphaleron) transitions convert it into baryon number. The expanding bubbles capture the non-vanishing baryon number and freeze it in by quenching the sphalerons, leading to $n_B \neq 0$ in the bubble interior. The baryon number is proportional to n_{left} , which in turn depends on CP-violating interactions and chemical potential equilibrating reactions such as Yukawa and nonperturbative $SU(3)_c$ transitions (strong sphalerons).

In this Letter, we reanalyze n_{left} in the context of the minimal supersymmetric Standard Model (MSSM), one of the best motivated conjectures of physics beyond the

SM, and observe new features which have been missed in previous works (see *e.g.* [5, 6, 7, 8, 9] and Refs. therein):

(1) Yukawa interactions between bottom quarks, Higgs bosons, and their superpartners cannot be neglected in EWB, even if the ratio of the vacuum expectation values (vevs) of the two MSSM Higgs doublets, $\tan\beta \equiv v_u/v_d$, is mildly larger than unity (a parameter region favored by current experimental constraints). This typically results in a qualitative change from the standard picture: the first two generations of quarks and squarks decouple in EWB if the first two generations receive CP asymmetry mostly through strong sphalerons, as is true in all well known scenarios. (2) the MSSM prediction for n_B/s can vary in magnitude and sign as the masses of the third generation sfermions are varied, even when the dominant source of CP violation is proportional to a single phase with a fixed sign. (3) there exist parameter regions in which left-handed (LH) leptons drive the baryon number production, unlike the traditional situations in which EWB proceeds mainly through the interactions of the LH quarks with electroweak sphalerons. This occurs in the large $\tan\beta$ region, in which τ Yukawa interactions are significant. Unlike standard thermal leptogenesis scenarios (see *e.g.* [3]), this new scenario does not require the participation of a right handed neutrino sector.

In what follows, we first present the computational framework and analytic intuition for our main results. We subsequently give the full numerical results and discuss their implications.

Framework and Analytic Intuition. The current density j_p^λ for each particle species p satisfies a quantum Boltzmann equation (QBE) of the form $\partial_\lambda j_p^\lambda = S_p^{\text{CP}} + S_p^{\text{CP}}$, where S_p^{CP} and S_p^{CP} are, respectively, CP-conserving and CP-violating source terms which depend on the MSSM interactions and chemical potentials. S_p^{CP} includes the terms that push the system toward chemical equilibrium, while S_p^{CP} contains the effects of CP-violating interactions involving the phase transition bubble. We have developed numerical solutions to the full set of QBEs for all MSSM particle species chemical potentials, including contributions to S_p^{CP} from previously neglected Yukawa and triscalar interactions as well as “supergauge” inter-

actions involving gauginos, particles, and sparticles.

The full numerical results, which we report at the end of this Letter, can be understood by considering an analytic solution which typically is valid in the limit of large $\tan\beta$ and superequilibrium which we explain shortly. In this regime, there exists a hierarchy of time scales in the phase transition dynamics that implies a set of simple relations between particle chemical potentials and densities: τ_{diff} , associated with the diffusion of particle densities ahead of the advancing bubble wall; a set of timescales τ_{eq} , associated with different interactions that move the plasma toward chemical equilibrium or zero chiral charge; and τ_{EW} , associated with the conversion of n_{left} into baryon number by the electroweak sphalerons. As we show below, typically $\tau_{\text{eq}} \ll \tau_{\text{diff}} \ll \tau_{\text{EW}}$, and the dynamics of the first and second generation (s)fermions largely decouple from those of the third generation, which then become the dominant source of n_B/s .

We begin with the largest time scales. It is well known that the electroweak sphaleron time scale is $\tau_{\text{EW}} \sim \Gamma_{\text{EW}}^{-1} \sim 10^5/T$, since $\Gamma_{\text{EW}} = 6\kappa\alpha_W^5 T$, with $\kappa \simeq 20$ [10], and α_W the $\text{SU}(2)_L$ analog of the fine structure constant. The diffusion time depends on an effective diffusion constant for the plasma, $\bar{D} \simeq 50/T$ [15, 17] and the velocity of the advancing wall, $v_w \sim 0.05$ [12]: $\tau_{\text{diff}} \equiv \bar{D}/v_w^2 \sim 10^4/T$ [14].

The following reactions determine τ_{eq} : a) for third generation fermions, Higgs scalars, and their superpartners, Yukawa interactions associated with the decay, absorption, and scattering of particles within the thermal plasma; b) strong sphalerons that favor a relaxation of chiral charge to zero; c) supergauge processes involving spontaneous emission and absorption of gauginos, such as $q + \tilde{V} \leftrightarrow \tilde{q}$. For example, the Yukawa-induced equilibration time-scale for third generation left-handed quarks (q), right-handed stops (\tilde{t}), and Higgsinos (\tilde{h}), driven by the scattering $q + \tilde{t} \leftrightarrow \tilde{h}$ is numerically $\tau_{\text{eq}}^{Y_t} \sim 20/Y_t^2 T$ where Y_t is the top Yukawa coupling for $m_{\tilde{h}} \approx 200$ GeV and $m_{\tilde{t}} \approx 100$ GeV. Since $Y_t \simeq 1$, $\tau_{\text{eq}}^{Y_t} \ll \tau_{\text{diff}}$, which in turn implies the approximation $\mu_q + \mu_{\tilde{h}} - \mu_{\tilde{t}} = 0$ on τ_{diff} time scales. Similarly, the time scale for strong sphaleron-induced relaxation of the total chiral charge, $N_5 \equiv \Sigma_j (2\mu_{q_j} - \mu_{u_j} - \mu_{d_j})$, where q_j , u_j , and d_j denote the left-handed quark doublet and right-handed quark singlets of generation j , is $\tau_{\text{eq}}^{\text{ss}} \sim \Gamma_{\text{ss}}^{-1} \sim 300/T$, since $\Gamma_{\text{ss}} = 6\kappa'(8/3)\alpha_s^4 T$ with α_s being the strong coupling and $\kappa' \sim \mathcal{O}(1)$ [11]. Hence, $\tau_{\text{eq}}^{\text{ss}} \ll \tau_{\text{diff}}$, leading to the condition $N_5 = 0$ on τ_{diff} time scales.

Finally, when the masses of gauginos (\tilde{V}) are sufficiently light, supergauge processes can lead to chemical equilibration between SM particles and their superpartners, a situation we denote as “superequilibrium” defined mathematically as $\mu_p = \mu_{\tilde{p}} \equiv \mu_P$. This implies

$$P \equiv p + \tilde{p} = (k_p + k_{\tilde{p}})\mu_p \frac{T^2}{6} \equiv k_P \mu_P \frac{T^2}{6}, \quad (2)$$

where k_p are statistical weights that relate charge density and chemical potential of particle species p : $p \equiv$

$j_p^0 = k_p \mu_p T^2/6$. Even when gauginos become heavy, supersymmetric Yukawa interactions can effectively bring about superequilibrium for third generation (s)fermions and Higgs(inos) in some regions of MSSM parameter space. The corresponding rates computed in [16, 17] indicate that typically $\tau_{\text{eq}} \ll \tau_{\text{diff}}$ for each supersymmetric three-body process. In the remainder of our analytic discussion, we will work in the superequilibrium regime where Eq. (2) holds. For example, we then have $\mu_{\tilde{t}} = \mu_t$ for (s)tops and $\mu_{\tilde{h}} = \mu_h$ for Higgs(inos).

Because the b quark and τ lepton Yukawa couplings are small compared to Y_t in the SM, they have been previously neglected in EWB computations. In the MSSM, however, they can be significantly enhanced by $\tan\beta$. The lower bound on the mass of the lightest supersymmetric Higgs boson and electroweak precision data such as the anomalous magnetic moment of the muon favor $\tan\beta \gg 1$. For $\tan\beta = 10$, for example, $Y_{b,\tau}^2/Y_t^2$ is 100 times larger than in the SM, suggesting the possibility of a relatively larger impact of the bottom and tau Yukawa interactions than in the SM [13]. After performing a numerical scan over the relevant MSSM parameters, we find that the bottom (tau) Yukawa-induced equilibration timescale is typically short compared to τ_{diff} for $\tan\beta \gtrsim 5$ (20) [17]. The Yukawa rates for the other generations of fermions are typically too slow compared to the diffusion rate to have a significant impact. In what follows, we will work in the regime where $\tau_{\text{eq}}^{Y_t}$, $\tau_{\text{eq}}^{Y_b}$, and $\tau_{\text{eq}}^{Y_\tau}$ are short compared to τ_{diff} and subsequently discuss possible departures from this domain.

The timescale hierarchy has several physical consequences some of which have already been noted. First, by the time particles have diffused well ahead of the advancing bubble, Yukawa-induced chemical equilibration and chiral charge relaxation have occurred. Second, for times $\lesssim \tau_{\text{diff}}$, total baryon and lepton number are approximately individually conserved, since their difference is always conserved and their sum is violated only the longer timescale τ_{EW} . Focusing first on the Higgs scalars and third generation fermions, we observe that Yukawa-induced chemical equilibrium and supergauge equilibrium implies to a good approximation that

$$\mu_q + \mu_h - \mu_t = 0, \quad \mu_q - \mu_h - \mu_b = 0, \quad \mu_\ell - \mu_h - \mu_\tau = 0 \quad (3)$$

where ℓ is the left handed lepton, the difference of the sign of μ_h in Eqs. (3) follows from conservation of hypercharge. Adding the first two implies that $2\mu_q - \mu_t - \mu_b = 0$, such that the third generation contribution to N_5 vanishes. As previously noted, on the time scale of τ_{diff} , we have $N_5 = 0$. Moreover, because the first and second generation Yukawa couplings (diagonal or off-diagonal in gauge eigenstate basis) are tiny compared to the diagonal values for the third generation, there exist no significant interactions to generate non-vanishing first and second generation quark densities. Consequently, the first two generation densities approximately vanish for large $\tan\beta$.

We now use the approximate conservation of baryon and lepton number on τ_{diff} time scale to relate the total

fermion plus sfermion densities to those for the Higgs plus Higgsinos. Since the first and second generation (s)quark densities vanish, the conservation of baryon number in terms of chemical potentials is $k_Q \mu_Q = -k_T \mu_T - k_B \mu_B$. Using $\mu_T + \mu_B = 2\mu_Q$ as implied by Eqs. (3,2), we obtain

$$\mu_Q = \frac{k_B - k_T}{k_Q + k_B + k_T} \mu_H \rightarrow Q = \frac{k_Q}{k_H} \frac{k_B - k_T}{k_Q + k_B + k_T} H. \quad (4)$$

Similarly, applying lepton number conservation and Eq. (2) to third generation leptons implies

$$L = \frac{k_L}{k_H} \frac{k_\tau}{k_L + k_\tau} H, \quad (5)$$

where L (τ) denotes the third generation left-handed (charged right-handed) lepton supermultiplet density. Using the approximate vanishing of first and second generation fermion densities we obtain the total left-handed fermion density

$$\begin{aligned} n_{\text{left}} &\simeq n_q + n_\ell = \frac{k_q}{k_Q} Q + \frac{k_\ell}{k_L} L \\ &\simeq \left[\frac{k_q}{k_H} \left(\frac{k_B - k_T}{k_Q + k_B + k_T} \right) + \frac{k_\ell}{k_H} \left(\frac{k_\tau}{k_L + k_\tau} \right) \right] H. \end{aligned} \quad (6)$$

Eq. (6) is the key analytic result for the large $\tan \beta$ superequilibrium regime. It relates the non-vanishing Higgs supermultiplet density induced by CP-violating transport dynamics to n_{left} via the statistical weights k_p for third generation SM fermions, Higgs bosons, and their superpartners. The first and second terms on the right hand side correspond to the contributions from third left-handed generation quarks and leptons, respectively.

The dependence on these contributions differs strikingly from what has appeared previously in the literature: $n_{\text{left}} \simeq 5Q + 4T$, with *e.g.*, $k_B - k_T \rightarrow k_B - 9k_T$ and $k_Q + k_B + k_T \rightarrow 9k_Q + k_B + 9k_T$ in the numerator and denominator, respectively, of Eq. (4). These differences arise from (1) the contributions from the LH third generation (s)lepton density engendered when Y_τ is sufficiently large, and (2) the Y_b -induced bottom quark superequilibrium conditions of Eqs. (3). As discussed above, including the latter implies vanishing third generation contribution to N_5 and a corresponding suppression of the first and second generation quark densities. Using our full numerical solutions, we have verified that in the regime of small Y_b and Y_τ , one recovers the previously identified dependence of n_{left} on Q and T .

The presence of the statistical weights in Eq. (6) implies a strong dependence of n_B/s on the masses of the third generation squarks and sleptons, as one may observe from the expression for the k_p :

$$\frac{k_p(m_p/T)}{k_p(0)} = \frac{c_{F,B}}{\pi^2} \int_{m_p/T}^{\infty} dx \frac{x e^x}{(e^x \pm 1)^2} \sqrt{x^2 - m_p^2/T^2}, \quad (7)$$

where $k_p(0) = 2g_p$ (g_p) for Dirac (chiral) fermions and complex scalars with degeneracy g_p , and $c_{F(B)} = 6(3)$

for fermions (bosons) and the $+$ ($-$) sign for fermions (bosons). For a fixed right handed (RH) stop mass, there exists a regime for $m_{\tilde{b}} \approx m_{\tilde{t}}$ in which $k_B \approx k_T$ in Eq. (6), corresponding to a nearly vanishing quark contribution to n_{left} . Indeed, this is the precise region in which the advertised sign flip can occur depending on the magnitude of sparticle masses. Furthermore, if the quark contributions are small, for sufficiently light $\tilde{\tau}_R$, EWB may actually be driven by the (s)lepton sector of the MSSM. This possibility does not arise under the previously studied assumptions of negligible $Y_{b,\tau}$ and represents a qualitatively new class of supersymmetric baryogenesis scenario.

Numerical solution and n_B/s . Before turning to full numerical examples, we first isolate the essential physics with an approximate, analytic solution. In the regime for which Q and L are proportional to H as in Eqs. (4,5), it is convenient to combine the QBEs into a single equation for H . Eliminating all terms containing the fast Yukawa and strong sphaleron rates, we find

$$\partial_\lambda j_H^\lambda = -\bar{\Gamma} \frac{H}{k_H} + \frac{S_H^{\text{CP}} + S_t^{\text{CP}} - S_b^{\text{CP}} - S_\tau^{\text{CP}}}{1 + K_T + K_L - K_B}, \quad (8)$$

where $\bar{\Gamma} = (\Gamma_h + \Gamma_T + \Gamma_B + \Gamma_\tau)/(1 + K_T + K_L - K_B)$ with $K_P \equiv H/P$, the Γ_P being chiral relaxation transport coefficients that vanish outside the bubble[7], and where the $S_{\tilde{p}}^{\text{CP}}$ denote the CP-violating source terms arising from the scattering of superpartners \tilde{p} from the space-time dependent Higgs vevs. The diffusion *ansatz* allows one express the LHS of Eq. (8) in terms of the density H : $\partial_\lambda j_H^\lambda = \dot{H} - \bar{D} \nabla^2 H$, where \bar{D} is the effective diffusion constant introduced earlier.

Here, we will rely on the popular Higgsino CP violating source (e.g. [5, 6, 9]) in order to illustrate the impact of the third generation sfermion masses and defer a study of possibly important S_b^{CP} and S_τ^{CP} contributions. For simplicity, we assume a common relative phase $\phi_\mu \equiv \text{Arg}(\mu M_i b^*)$ ($i = 1, 2$), where b is the SUSY-breaking Higgs mass parameter. As a benchmark, we work in the resonant regime and choose $|\mu| = |M_2| = 200$ GeV and $|M_1| = 100$ GeV, computing S_H^{CP} and $\bar{\Gamma}$ from the expressions in Ref. [7] and a bubble wall profile given in [6] for $\tan \beta = 20$. We consider a scenario with a light RH stop with $m_{\tilde{t}_1} = 150$ GeV as needed to obtain a strong, first order phase transition. All first and second generation sfermions and third generation LH sfermions have masses equal to one TeV.¹

The results for n_B/s as a function of the RH bottom squark mass, $m_{\tilde{b}_1}$, in units of the WMAP central value for $\sin \phi_\mu = -1$ are given in Fig. 1. The solid and dashed curves give the results obtained with the complete numerical solution of the QBEs and the large $\tan \beta$ ana-

¹ The mass $m_{\tilde{t}_1}$ denotes the RH top squark mass at $T = 0$, after EWSB, while $m_{\tilde{t}}$ is the finite T mass in the unbroken phase; similar notation applies for sbottom and stau as well.

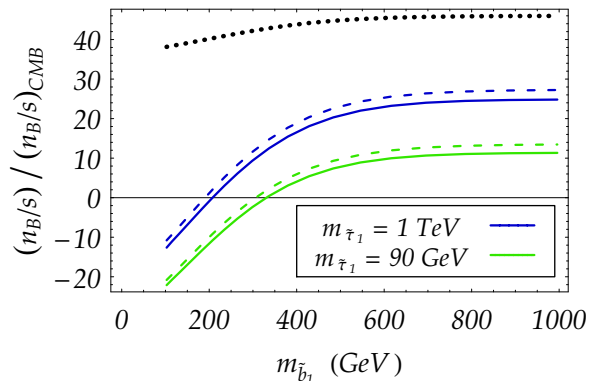


FIG. 1: Baryon asymmetry in units of the WMAP central value as a function of RH bottom squark mass. The two solid (dashed) curves were obtained using the full numerical (large $\tan\beta$ analytic) solution. Light green (dark blue) curve corresponds to RH stau mass of 90 GeV (1 TeV). The dotted curve gives results neglecting bottom and tau Yukawa interactions.

lytic solution embodied in Eqs. (6,8), respectively, for two representative RH tau slepton mass: $m_{\tilde{\tau}_1} = 90$ GeV (light green) and one TeV (dark blue). The dotted curve shows the numerical result obtained when bottom and tau Yukawa interactions are neglected.

The impact of including realistic bottom and tau Yukawa interactions in the large $\tan\beta$, superequilibrium regime is striking, as seen in Fig. 1. Neglecting $Y_{b,\tau}$ generally leads to a baryon asymmetry that is larger in magnitude, positive, and relatively insensitive to $m_{\tilde{b}_1}$. The close agreement between the solid and dashed curves indicates our foregoing analysis captures the primary features of the dynamics in this regime. As expected, n_B/s is largest in magnitude when $m_{\tilde{b}_1}$ is heavy and decreases as $m_{\tilde{b}_1}$ is decreased, reflecting the growing importance of sbottoms as they become light and the greater cancellation between k_T and k_B in Eq. (6). For heavy RH staus, the magnitude of the (s)lepton contribution to n_{left} is small, and so n_B/s vanishes when $m_{\tilde{b}} \sim m_{\tilde{t}}$ and $k_B \sim k_T$.

For light staus, the (s)lepton contribution becomes important, and for either very heavy or very light sbottom, this contribution can change n_B/s by a factor of two. (Since $n_B/n_{\text{left}} < 0$, very light \tilde{b} and $\tilde{\tau}$ lead to negative n_B/s , and for large $m_{\tilde{b},\tau}$, n_B/s is positive.)

Deviations from Eq. (6) can occur for small $\tan\beta$ or if superequilibrium is not valid. The region of $\tan\beta \lesssim 10$ corresponds to an interpolation between the upper dashed and the dotted curves in Fig. 1, reflecting the larger third generation quark and smaller leptonic components to n_{left} . Although we have estimated that the off-diagonal CKM matrix elements are sufficiently small to justify neglect of flavor-changing $2 \rightarrow 2$ scattering processes, such corrections should be kept in mind. We have also made (standard) assumptions regarding the trilinear scalar couplings which affect the decoupling of the light generations. Exploration of these and other effects will appear in forthcoming publications [17].

Summary. During the upcoming era of more sensitive electric dipole moment searches and LHC studies, it is crucial to explore testable EWB scenarios such as MSSM baryogenesis [4]. We have shown in this context that the bottom Yukawa coupling is important even for moderate values of $\tan\beta$, which leads to dramatic changes in the basic physical EWB mechanism and the associated MSSM parameter space constraints. We also showed that the magnitudes of the sparticle masses can change the sign of the baryon asymmetry, and identified a new lepton driven supersymmetric baryogenesis scenario which does not involve RH (s)neutrinos.

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